Developing Mathematics Achievement and Inductive Reasoning: A Proposed Technique According to Brain Compatible Learning Theory

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Recent research has shown the need to present knowledge to students in a manner consistent with the characteristics of the human brain, especially during learning mathematics, which contributes to the development of achievement and inductive reasoning. The current study aimed to develop achievement and inductive reasoning by employing brain-compatible learning theory in teaching and learning mathematics for sixth-grade students in Oman. The sample included (74) students, who were divided into two equal groups: experimental and control. Study data were collected using mathematical achievement tests and inductive reasoning. There were significant differences between the two groups in the post-application of the achievement and inductive reasoning tests in favor of the experimental group, which indicates that teaching mathematics according to the theory of learning compatible with the brain had a significant impact on the development of achievement and inductive reasoning among the participants. The study recommended the need to develop and teach mathematics curricula in a way that allows for the practice of many activities and exercises that contribute to the development of students' achievement and inductive reasoning skills.

Keywords: learning theory, brain compatible, achievement, inductive reasoning

INTRODUCTION

Research on the structure of the human brain has evolved and diversified in recent years, focusing on the necessity of providing scientific content to students consistent with the structure and characteristics of the human brain (Ryan, 2018). It also focused on developing students' minds and enabling them to acquire to imagine, deduce, invent, create and criticize (Pape et al., 2003) to achieve the best possible results for teaching and learning processes.

Therefore, there should be an emphasis on developing different types of thinking, and enhancing the student's abilities to organize, develop, and employ the mental processes of the gained experiences and methods of processing them (Drijvers et al., 2019). Thinking is a goal always present in mathematics education curricula at all levels of education (NCTM, 2014; Elsayed, et al., 2021 a).

Learning mathematics is an excellent way to achieve thinking for all learners, as it relies on imagination, employing the mind, building correct mental images, and providing supporting evidence for different situations. Mathematics is also distinguished by special features that make it unique from other academic subjects, as it is characterized by a practical, experimental, cumulative, synthetic, and deductive nature simultaneously (Aloufi et al., 2021; Elsayed, 2015a).

Dascalu (2012) mentions that thinking about mathematics helps students succeed in their real life because it enables them to solve their practical problems and develop various abilities such as research, investigation, and experimentation.

The National Council of Teachers of Mathematics (NCTM, 2015) indicates that thinking in mathematics is a new direction in its teaching and learning, and the development of mathematical thinking has become a necessity nowadays, given global developments. For this reason, teaching mathematics has evolved from focusing on knowledge, procedures, and problems to complementing mathematical thinking with its various skills and increasing general understanding of mathematics, along with teaching mathematics for life (William et al., 2009; Elsayed, et al., 2021 a).

Inductive reasoning is one of the cognitive processes that begins with working with specific special cases and leads to the formulation and verification of conjectures. It is one of the basic types of thinking introduced in the early years of education to help students from an early age to acquire knowledge (Polya, 1957). All countries worldwide have developed curricula and technologies to enable students to learn ways of thinking in a way that contributes to the development of students' minds and direction toward mathematics. In this way, they can overcome the local and global problems confronting them in different aspects of their lives (Irwin & Britt, 2005; Doerr, 2006; Rushton, 2006; Drijvers et al., 2019).

As Academic achievement and inductive reasoning are important goals of education experts and educators are indicating to use of the recent theories and strategies that increase these skills for learners at different levels of education (Palatnik & Koichu, 2017; Chrysostomou et al., 2013). Also, it becomes important to look for methods and teaching strategies that achieve greater interest to benefit from the research that describes how the brain worked to become a teaching and learning accessible and acceptable to students and commensurate with their thinking and concerns properties, where the results of some studies have confirmed that the teacher training on brain-compatible learning theory has helped their students to increase achievement and thinking in mathematics (Gocmen & Coskun, 2019).

Rindermann (2018), through his analysis of the results of international studies on school achievement and reasoning in mathematics, states that there are significant differences between countries in those variables. Reviewing the current situation in Oman concerning academic achievement and inductive reasoning skills, we find that the average of Omani students in international tests (TIMSS, 2018) reached 416 points, which is lower than the global average in those tests of 500 points (Omani Ministry of Education, 2022). Also, the study of (Elsayed & Albarami, 2019) conducted on (200) learners in the six grade in Oman revealed that the performance of these learners on the questions of higher levels in the final mathematics tests was significantly weak. Therefore, it can be said that the six grade students in Oman have inadequate mathematics skills and poor inductive reasoning.

Brain Compatible Learning Theory

The brain-compatible learning theory in the late twentieth century (Ryan, 2018) confirms that students can learn effectively. His learning skills improve if the learning environment available to him is an active environment compatible with how his mind works (Laura, 2014), as this motivates learning by allowing learners to interact appropriately with the educational experiences (Tara, 2016). Ryan (2018) mentions that the brain compatible learning is mainly concerned with the mind, the brain, and education, that is, the mechanism of the mind (the thinking process itself) with the brain (the organ in charge of the thinking process) and how this reflects on education. It is learning based on following a comprehensive approach based on research in neuroscience and preparing the brain for learning naturally (Laura, 2014). It also represents a framework for teaching and learning, helps explain repetitive learning behaviors, and assures teachers of teaching students real-life experiences (Spears & Wilson, 2012; Elsayed, 2015 b).

Some Experts (Caine & Caine, 2002; Gozuyesil & Dikici, 2014; Jeffery, 2002) identified some principles of brain-based learning as: Simulating the environment in terms of color and structure so the presentations should be the responsibility of the students not the teacher and thus the links are formed in the brains of the students; Identifying places for learning in groups and social learning to simulate the social requirements of the brain; Achieving connection between indoor and outdoor locations, involving the cerebral motor cortex to increase the oxygen flow; Corridors and places in the school must contain explanatory signposts that indicate the school community; Diversification of display screens and ways of interacting with environment so as to achieve brain simulation; Caring of the former learning to achieve flexibility in providing materials for students; Providing opportunities for independent self-reflection away from others; Providing optimal environment for learning to take the full advantage of the environment such as technology components and distance learning.

In this regard, (Elsayed, 2015 b; Jensen 2008; Laura, 2014; Ryan, 2018) identified the characteristics of the learning environment in light of brain-compatible learning as follows: The Learning Environment should be out of tension for the student because there is an inverse relationship between the physical environment that raises anxiety and the failure of the student and his concentration; The Learning Environment should be out of threat for the student (the threat and learning), students who are exposed continuously to the threat and tension of a high rate in the early stages of their childhood have got a behavior away from teaching and the learning process; The Learning Environment must not be suggestive that the student is stupid, as this impairs his determination to learn so that the teacher must reduce the circumstances that cause this feeling and uses personal strategies to teach students such as Time Management, Games and Diverse Discussions Strategies.

Therefore, many attempts have been made to develop teaching models and strategies based on the brain-compatible learning theory. Mariel (2013) has provided a model for teaching directed to the brain. It describes the six stages of the process of learning and teaching. They provide an appropriate emotional atmosphere for learning, create a comfortable physical learning environment, design a successful learning experience, efficiently teach procedural knowledge and judgment, teach the application of knowledge well, and assess learning appropriately. Even though these stages are studied separately, these stages have an internal connection. Jensen (2008) provided a strategy consisting of three primary stages, each including specific steps. They are Pre-Learning Stage: It represents 10% of the time of the teaching process. It includes the preparation and creation of the learner and provides the optimal environment for learning; during the learning process Stage: It represents 80% of the time of the teaching process.

Many authors identified some teaching methods that fit the idea of Brain-based learning identified some teaching methods that provide the idea of the Brain-based learning such as Direct Teaching, Jigsaw Strategy, Demos Method, Concepts Reaching, Socratic Method, Cooperative Learning, Simulation, Role Play, Individual Learning Methods, Brain Storming Strategy, Mind Maps Strategy, Interactive Teaching Strategy and Six Hats Strategy (Caine & Caine, 2002; Elsayed & Abbas, 2021b; Gozuyesil & Dikici, 2014; Jensen, 2008; Sousa, 2009).

Inductive Reasoning

Inductive reasoning is precisely among the types of mathematical thinking to reveal regularity or lack thereof by forming rules and generalizations (Klauer & Phye, 2008). It is improved by developing students' abilities to solve problems, reason and logical thinking, and present mathematical topics interestingly and enjoyably for students while making the application the focus of the educational process (Reyes & Amarnani, 2015).

Inductive reasoning is based on moving from the parts to the totals and ensuring that a sure thing applies to the parts of the phenomenon. In inductive thinking, dealing with the entire research community, none of its elements is excluded to come out with results that can be trusted (Torres et al., 2021).

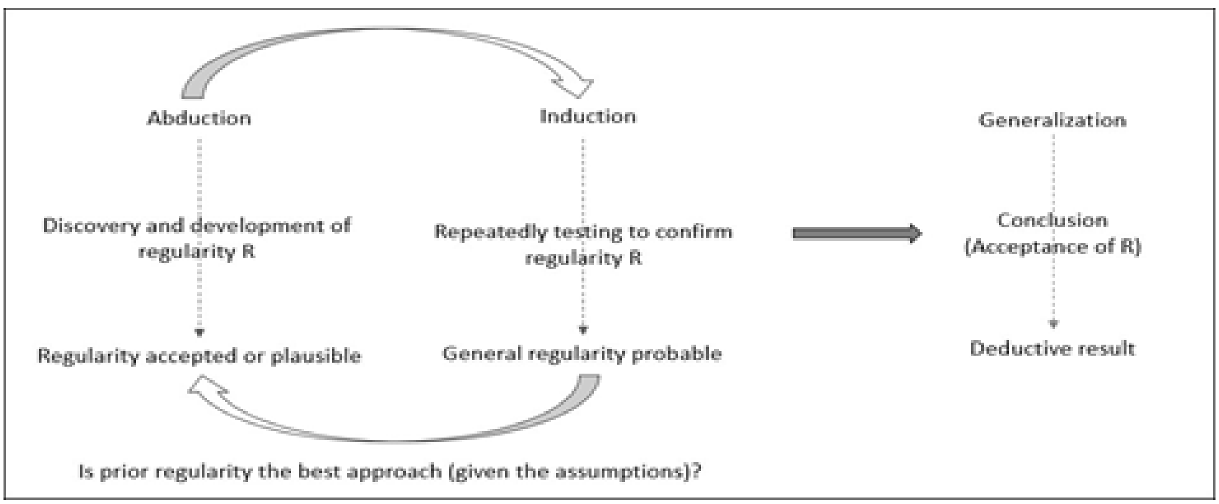
Inductive reasoning is the appropriate way to predict different learning outcomes, in addition to the student's ability to use mental processes in solving mental problems through classifying information, developing dimensions for inferences, determining relationships, building concepts, solving problems, developing hypotheses and testing them (McGrew, 2009).

Like induction, abduction has been presented as a form of inference characterized by hypothesis generation. It is the first and most miniaturized stage of private reasoning, as it entails the construction of preliminary explanations (Nepomuceno, 2005). Abduction may be a lower cognitive state than inductive reasoning (Nino, 2012), and induction may not consist of creativity but verification; in other words, inductive reasoning does not introduce new knowledge (Torres et al., 2021). Potential errors are expected to be corrected at the induction stage, which progresses towards generalization based on the available evidence. Induction can therefore be considered as a powerful knowledge-construction resource that mainly owes its potential to the presence of generalization as one of its main components (Rivera & Becker, 2007).

Generalization is a key component of inductive reasoning and is also essential to mathematical reasoning, which entails seeing beyond the idiosyncrasies of a mathematical situation to infer it (Driscoll, 1999). Students' practice of generalization makes them able to distance themselves from the details inherent in computational computations and thus identify the structure and mathematical relationships involved (Elsayed et al., 2021a).

Rivera and Baker (2007) conducted a study of patterns of abduction that precede generalization. During collective inference, subjects discover regularity, which makes them able to make initial guesses when working with the cases involved in a given task. In the further stage, the structure is accepted as valid based on such early special cases and is subjected to validation with more problems, eventually leading to generalization. A conclusion drawn from collective inference carries less weight than a conclusion drawn from induction (Aguayo, 2011). Induction requires confirmation of the guess given. Regularity is accepted after confirmation, allowing the subject to generalize (Rivera and Baker, 2007). See Figure 1.

FIGURE 1
ADDUCTIVE-INDUCTIVE PROCESS



Rivera, 2007

Therefore, the current study seeks to employ the learning theory compatible with the Brain in teaching mathematics to develop academic achievement and inductive reasoning in Omani students in the sixth grade.

In light of previous theoretical literature, the foundations of teaching mathematics based on the brain-compatible learning theory to development academic achievement and inductive reasoning can be identified as follows: 1) Teaching/ Learning Physical Environment consists of Diversifying the physical environment of the classroom by adding educational panels, changing the seating and forming groups, providing an atmosphere of intimacy and friendliness in the classroom, Ensuring the safety of lighting in the school, Diversification of educational methods used in the lessons, using some decorative works related to mathematics and produced by students themselves; 2) Teaching/ Learning Scientific Environment consists of Linking previous relevant information with the new information in lessons, Diversifying stimuli learning and participation of students, Diversifying teaching methods (Learning in Groups, Dialogue, and Discussion, Brainstorming), Working on Fragmentation of information when it is presented for the first time, Providing optional opportunities for pupils, Training students to conclude and create, Providing opportunities for linguistic expression (orally or in writing) for some information in the lesson, Using technological means as much as possible, Promoting the correct answers to students, Encouraging Cooperative Learning, Encouraging risk to clarify that we can learn by making mistakes; 3) Evaluating students in learning the lesson consists of Giving attention to immediate feedback during the lesson, evaluating individual students within the group, Positive reinforcement to the results of the performance of pupils in groups as well as individual performance of each student, diversifying methods of performance: (Oral, Writing, Individual, Collective), Identifying a variety of home assignments, Developing and follow a fixed reward and punishment rules during the lesson.

As for the steps of teaching mathematics according to the brain-compatible learning theory for the development of achievement and inductive reasoning, it can be determined as follows: 1) Initialization: The logical inputs and mathematical correlations are created between previous learning experiences and the subject matter to prepare the student's brain to receive new information in a meaningful sense. It is provided by making mental images of relationships and concepts related to the content of the new lesson and revising them; 2) Acquisition: In this stage, the learner gains an acquisition. He got the new lesson by achieving logical nervous linkages consequences due to the link between the preceding and familiar information with the further information by using a variety of connected stimuli between the learner and the use of Cooperative and Collective Learning experiences and Discussion with the group and the rest of the other groups; 3) Expansion: This is to attract students to participate, sign up, and form the design of learning experience and acquisition to deepen and expand learning through Trial and Error, Feedback, Communication, and Enhance Memory; 4) Evaluating student learning: It specifies the academic achievement of the lesson's learning objectives. It requires diversification of methods of performance. It is determined by (Oral, Writing, Individual, and Collective), and there should be a variety of home assignments.

METHODS AND MATERIALS

Research Model

We used a quasi-experimental design of two independent experimental and control groups, which were randomly selected. The students in the experimental group received the proposed unit in mathematics based on brain-aligned learning theory, and the students in the control group received the same unit using the way schools see it as a teacher. Finally, we examined whether the experimental group's students outperformed the control group's students on mathematics achievement and inductive reasoning. The independent variable of the current study is the teaching style used, either teaching mathematics according to the learning theory compatible with the brain or the traditional method. Dependent variables are an academic achievement in mathematics and inductive reasoning.

Participants

The study sample consisted of (74) students in the 6th grade from Khalid bin Al-Walled school in Salalah. The students were randomly divided into an experimental group (37) and a control group (37).

Mathematics Teacher's Manual

The authors have built a manual for mathematics teachers to teach the third unit included in the Grade 6 book (the Geometry unit) using brain-aligned learning theory. This manual consisted of many elements: unit objectives, aspects of learning, inductive thinking skills, instructional methods and activities, assessment methods and styles, and evaluation steps. The manual's suitability was verified by submitting it to seven experts in mathematics education in schools, and it was prepared in its final version based on their various directions.

Academic Achievement Test

The authors built the academic achievement test in mathematics so that the first draft of it consisted of (25) multiple-choice questions in the geometry unit of the sixth-grade textbook in Oman.

The authors built a test of the mathematics academic achievement in the geometry unit of the sixth-grade textbook in Oman, which consisted of (25) multiple-choice questions. This test has been validated and reviewed by eight experts in mathematics education. Some items have been deleted, and others have been modified based on experts' suggestions, so that it consists of (20) questions. Test reliability was measured by application on a sample of (40) students from East Salalah School as a pilot study. The value of the stability coefficient, according to Guttman's equation, was (0.88). This value indicates that the test has a significantly high level of reliability. We want to point out that the maximum score for the test was (20), and the minimum score was (0). So, one point was given for the correct answer to each question and zero points for the wrong answer. The appropriate time to apply the test was (40) minutes, according to the average response times of students to the test in the pilot study.

Inductive Reasoning Test

The authors built a test of inductive thinking skills in the geometry unit of sixth-grade students in Oman, which consisted of (25) multiple-choice questions distributed equally over the five inductive reasoning processes (generalization, discrimination, cross-classification, perception of relationships, and derivative relationships) based on relative importance for each skill as well as the number of sub-skills for them. The test was validated by being reviewed by eight experts specializing in teaching and learning mathematics and then modifying some of its elements based on their suggestions. To ensure the reliability of the test, it was applied to a sample of (40) students from East Salalah School as a pilot study; then the reliability coefficient was calculated using the Cronbach equation. The Cronbach's alpha reliability value was (0.88), meaning the test has a high level of reliability. The maximum score for the test was (20), and the minimum score was (0), so one point was given for the correct answer to each question and zero points for the wrong answer. The appropriate time to apply the test was (45) minutes, according to the average response times of students to the test in the pilot study.

Data Collection Process

The study experiment was carried out for (7) weeks in the first semester during the academic year 2021/2022, so that teaching takes place four times a week. Thus the total number of teaching sessions for the unit is (28) lessons. The control and experimental groups were tested, then the experimental group was taught by the mathematics teacher at Khalid Bin Al Waleed School using the brain-compatible learning theory. At the same time, the control group was taught by the same teacher using the traditional technique. Finally, at the end of the study experiment, the students of both groups were post-tested.

Data Analysis

Data were analyzed using means, standard deviations, t-tests, significance levels, and $\eta 2$, based on the SPSS 22.0 software package. We used the previous methods because we found the distribution to be normal. After all, the arithmetic values of the mean peak value of the obtained scores are close and because normality tests (Kolmogrov-Smirnov and Shipiro Wilk) have significance values of $p > 0.05$. In this regard, Grissom and Kim (2012) recommend the use of the SD pretest for students of both groups. It is not recommended to collect the four SDs for both pretest and posttest for both students of the two groups, because the experimental treatment often increases variance, leading to divergent values of SD for posttests for students of the experimental and control groups.

Pre –Test Application

To check the equivalence of the students in the experimental group and the students in the control group on the pretests of the mathematics academic achievement and inductive reasoning tests in the pre-test, T test has been used, as shown in table 1.

TABLE 1
DIFFERENCE BETWEEN THE MEANS OF THE TWO GROUPS ON THE PRE-TESTS OF THE ACADEMIC ACHIEVEMENT AND INDUCTIVE REASONING

	Experimental Group			Control Group			T	p
	N	Mean	SD	N	Mean	SD		
Achievement	37	3.29	1.77	37	3.21	1.53	0.24	1.46
Inductive Reasoning	37	4.63	2.34	37	5.08	2.20	0.99	1.11

Table 1 shows that the differences between students of the experimental and control groups in the pre-application, without exception, are small and insignificant. Hence, there is significant equivalence between the students of the two groups in academic achievement and inductive reasoning. This suggests that we can simplify us analyzes by comparing the post-test scores of students of the control and experimental groups.

RESULTS/DISCUSSION

The current study seeks to employ the brain-compatible learning theory in teaching mathematics to develop academic achievement and inductive reasoning at Omani students in the sixth grade. Therefore, we presented steps to employ this theory in teaching and learning mathematics for Oman's sixth-grade students at the end of the introduction and literature review. Also, to find out the effectiveness of this technique in developing the mathematics achievement and Inductive Reasoning of the sixth-grade students in Oman, we verified the following: The students of the experimental group outperformed the students of the control group in the post-application of each of the mathematics achievement and thinking tests. Tables 2 and 3 illustrate this.

TABLE 2
DIFFERENCE BETWEEN THE MEANS OF THE TWO GROUPS ON THE POST-TEST OF THE MATHEMATICS ACHIEVEMENT

Academic Achievement	Experimental Group			Control Group			t	p	η^2	ES
	N	Mean	SD	N	Mean	SD				
	37	18.20	0.99	37	11.15	0.79	3.96	0.001	0.18	0.93

ES denotes the Effect of Size

Table 2 shows very vast significant differences between the groups in academic achievement in favor of the experimental group. Employing the brain-compatible learning theory in teaching mathematics led to the development of the academic achievement of the study sample students. It is worth noting that the effect size of this technique was high (0.93), as we used 2η to measure the effect size of the independent variable on the dependent variable, which is a bidirectional correlation between the dependent variable and the independent variable (Winer et al., 1991).

Table 3 shows very vast significant differences between the groups on the inductive reasoning skills in favor of the experimental group. Employing the brain-compatible learning theory in teaching mathematics led to the development of inductive reasoning skills on all individual skills and the total score of the study sample students. It is worth noting that the effect size of this technique was high for all individual measures and the total score, as the effect size values vary from 0.90 to 2.05. This means that the effect size of employing the brain-compatible learning theory in teaching mathematics was highly significant for all inductive reasoning skills in isolation and the skills combined altogether.

TABLE 3
DIFFERENCE BETWEEN THE MEANS OF THE TWO GROUPS ON THE POST-TEST OF THE INDUCTIVE REASONING

Inductive Reasoning Skills	Experimental Group			Control Group			t	p	η^2	ES
	N	Mean	SD	N	Mean	SD				
Generalization	37	3.59	1.98	37	2.38	1.61	3.77	0.001	0.18	0.93
Discrimination	37	3.55	0.83	37	2.29	1.07	3.65	0.001	0.17	0.90
Intersecting Classifications	37	5.30	0.88	37	3.83	1.52	3.98	0.001	0.19	0.97
Perceiving Relationships	37	5.24	0.87	37	3.75	1.16	3.88	0.001	0.18	0.93
Deriving Relationships	37	1.68	1.02	37	0.85	1.22	3.55	0.001	0.17	0.90
Total	37	19.36	4.33	37	13.10	3.39	6.60	0.001	0.39	2.05

CONCLUSION

Employing the brain-compatible learning theory in teaching and learning mathematics significantly impacted the development of academic achievement and inductive thinking skills among the students of the study sample. Some of the reasons were discussed: 1) Employing brain-compatible learning theory contributed to linking students' different knowledge and information with each other. This, in turn, may have helped them understand the relationships between knowledge and practice of inductive reasoning skills, such as generalizing, cross-classifying, discriminating, perceiving relationships, and deriving relationships. 2) Employing the proposed theory reinforced students' motivation to make more efforts to achieve excellence and creativity in their various tasks while highlighting the spirit of competition among them in the classroom. 3) Students' practices of inductive reasoning activities have led to maximizing their roles within the school in general and the classroom in particular, which provided them with many opportunities to achieve excellence and creativity while practicing different creative solutions and methods to answer questions. 4) Employing the theory of brain-compatible learning in teaching mathematics among sixth-grade students helped them break the barriers of the prevailing routine in the educational process urging and motivating them to be active and effective in their participation and thinking, and continuously attracting their attention inside the classroom. 5) The proposed employment encouraged students to participate in sports competitions inside and outside the school, which increased their ambition and continuous desire to accomplish more activities and tasks. 6) To use the proposal, help the students organize their knowledge and experiences gained in the classroom, improve their knowledge awareness, and involve them in building their own knowledge structures. 7) Employing brain-compatible learning theory succeeded in taking into account individual differences among students, which increased their motivation for learning, achievement, and competition among them. 8) Employing this theory helped students increase their motivation and encouraged them to participate in all mathematical tasks and activities without exception while employing their various potentials and abilities, relying on the students' realistic environment as a rich source of learning.It should also be noted that many studies have found that academic achievement and inductive reasoning can be developed when teaching math through other methods and models, such as: (Chrysostomou et al., 2013; Doerr, 2006; Elsayed, et al., 2021 a, Elsayed & Abbas, 2021 b; Palatnik & Koichu, 2017; Reyes & Amarnani, 2015).

Despite the impressive results of the study, it can be seen that the sample size is somewhat limited but reasonable when considering the amount of time spent teaching students. Many training studies and experimental treatments show much smaller sample sizes. It should also be noted that the students in the study sample did not come from the rich regions of the Sultanate of Oman but rather from one of the central regions of the Sultanate. Therefore, some may find it difficult to generalize the results to all regions of Oman. However, it may be pointed out that when applying an approach or theory is successful in a poor area with a relatively low investment in education, it is necessarily successful in rich areas with a high investment in education.

RECOMMENDATIONS/FUTURE DIRECTIONS

The authors recommend several recommendations and proposals: 1) The need for experts to develop mathematics curricula in Oman to focus on preparing curricula that include exercises and activities that make students able to practice inductive thinking skills instead of focusing on memorizing and memorizing. 2) Reconsider expanding the scope of employing the brain-compatible learning theory to teach all mathematics courses at different levels of education. 3) Allowing in-service mathematics teachers to benefit from the evidence prepared by this study, with the need to train them to employ modern approaches and theories in teaching mathematics, such as the learning theory that is compatible with the brain, to develop their thinking skills and that of their students. 4) Comparative studies should be done on the effect of using learning theory compatible with the brain and other different theories and approaches on the development of many variables, skills, and skills.

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